DASC 2594\_Multivariable Math for Data Scientists\_Unit #2\_ Vector Spaces, Eigen decompositions, Orthogonality, Symmetry, and Quadratic Forms

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| **Stage 1 Desired Results** | | |
| ESTABLISHED GOALS  **Timeframe:** (maximum) 18 days  **Lesson Plans Referenced**:   * DASC 2594\_Unit 2\_Lesson Plan 4\_Vector Spaces, Rank, Dimension, and Bases * DASC 2594\_Unit 2\_Lesson Plan 5\_Eigenvectors and Eigenvalues * DASC 2594\_Unit 2\_Lesson Plan 6\_Orthogonality and Least Squares * DASC 2594\_Unit 2\_Lesson Plan 7\_Symmetric Matrices and Quadratic Forms | ***Acquisition*** | |
| *Students completing Unit 2 of DASC 2594 should be able to:*   * Define and apply the rank, basis, and invertible matrix theorem * Understand the definition of a vector space in and the properties of vector spaces * Identify vector subspaces and minimal spanning sets * Apply eigenvalue decompositions and explain how the eigen decomposition is a change of basis * Diagonalize a matrix and explain the relationship between diagonalization and linear independence * Apply the orthogonal projection theorem * Evaluate whether two vector spaces are orthogonal * Apply Gram-Schmidt orthogonalization to obtain an orthogonal basis to generate a QR factorization * Apply least squares to solve linear models | *Students will be skilled at…*   * Applying Eigenvalues and troubleshooting why software fails to calculate Eigenvalues * Identifying when systems of equations have solutions * Constructing orthonormal bases from matrices * Solve linear models using least squares and QR factorization * Solving the characteristic equation det(A – lambda I) = 0 to determine if lambda is an eigenvalue * Applying inner products, norms, and vector lengths to demonstrate orthogonality and angles between vectors * Identifying and correcting matrix decomposition errors due to numeric underflow |
| ***Meaning*** | |
| UNDERSTANDINGS  *Students will understand …*   * The rank of a matrix/dimension of and understand how rank is related to a basis * How matrix nullspaces, columnspaces, rank, and invertibility are related * Change of bases * Matrix determinant relates to area/volume of a matrix transformation * Understand coordinate systems and apply change-of-coordinate transformations * Understand the benefits of orthogonal bases in solving least squares | ESSENTIAL QUESTIONS   1. What is a vector space and what is a vector subspace? 2. What is the relationship between the null space, the column space, and linear transformations? 3. What is a basis and how to we transform between basis representations? 4. What is meant by dimension of a vector space? How is this related to matrix rank and linear independence? 5. What are Eigenvalues? 6. Why are Eigenvalues important in data science? 7. What do Eigenvalues tell us about the behavior of dynamical systems? 8. What is meant by the angle between vectors? What is meant by magnitude? 9. What is an orthonormal basis? How is an orthonormal basis different from other bases? 10. How are orthonormal bases used to solve problems in data science? 11. What is the relationship between symmetric matrices and diagonal forms? 12. What is a quadratic form? 13. What is the principal axis of a quadratic form 14. How can quadratic forms be used to characterize matrices? 15. How can you solve constrained optimization problems using quadratic matrices 16. How is the singular value decomposition used in data compression and how is it related to principal component analysis |
| ***Transfer*** | |
| *Students will be able to independently use their learning to…*   * Troubleshoot data science calculations – particularly those relating to singular value errors * Identify if a matrix decomposition (Eigen decomposition) fails due to lack of positive definiteness or numeric underflow * Check if a matrix is full rank * Apply Eigen decompositions to positive definite matrices to identify the primary coordinates of variation * Solve least squares problems using Eigen decompositions and QR decompositions | |
| **Stage 2 - Evidence** | | |
| **Evaluative Criteria** | **Assessment Evidence** | |
| Rubric Names: | PERFORMANCE TASK(S)    *Pre-Test:*  *Formative Assessment :*   * *HW 05: Vector Spaces* * *HW 06: Rank, dimension, change of basis, and Eigenvalues* * *HW 07: Eigenvectors and Eigenvalues* * *HW 08: Symmetric Matrices, Quadratic Forms, and the Singular Value Decomposition*   *Summative Assessment:*   * Exam (In class and take home) | |
|  | OTHER EVIDENCE:   * In class questions using learning software (google forms, etc.) * Student feedback and questions | |
| **Stage 3 – Learning Plan** | | |
| *Summary of Key Learning Events and Instruction*  *Unit #2 (maximum) 18 days*  Textbook: To be determined (likely will be primarily based on class lecture notes)  Technology: RStudio /RStudio Server / RStudio Connect;  Prerequisite Knowledge/Course(s): Fundamental understanding of Calculus at the level of Calculus II and ability to program in R  **Activity 1 (6 days):** List Topic Refer to DASC 2594\_Unit 2\_Lesson Plan 4\_Vector Spaces, Rank, Dimension, and Bases  **Activity 2 (5 days):**  List Topic Refer to DASC 2594\_Unit 2\_Lesson Plan 5\_Eigenvectors and Eigenvalues  **Activity 3 (4 days):**  List Topic Refer to DASC 2594\_Unit 2\_Lesson Plan 6\_Orthogonality and Least Squares  **Activity 4 (3 days):**  List Topic Refer to DASC 2594\_Unit 2\_Lesson Plan 7\_Symmetric Matrices and Quadratic Forms | | |

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| **Learning Accommodations** | |
| **Student Accommodations** | **Accelerated Students** |
| Compliance/ADA/504 | Challenge Students who want/need more |